Comparison of two methods for the finite element analysis of nonlinear 3D periodic eddy-current problems

René Plasser[1], Yasuhito Takahashi[2], Gergely Koczka[3] and Oszkár Bíró[1], *Member, IEEE*

[1]IGTE, Graz University of Technology, 8010 Graz, Austria, rene.plasser@tugraz.at
[2]Department of Electrical Engineering, Doshisha University, Kyoto 6010-0321, Japan, ytakahashi@mail.doshisha.ac.jp
[3] Transformers Weiz, Siemens Inc. Austria, 8160 Weiz, Austria, gergely.koczka@siemens.com

Two different techniques for the analysis of nonlinear, periodic eddy-current problems are compared using a 3-dimensional benchmark problem. The methods are the parallel time periodic-finite element method and the harmonic balance fixed-point technique.

Index Terms **—Eddy currents, nonlinear magnetics, finite element analysis, computational electromagnetics**

I. Introduction

The topic of this investigation is to validate various methods for the finite element analysis of nonlinear, 3-dimensional eddy-current problems in which steady state solutions are of interest. For this reason a basic single phase transformer enclosed by a steel tank has been modeled. On each limb of the core there is half of the primary and secondary winding. To be as close to a practical application as possible, the transformer primary winding is voltage driven. Due to the highly permeable materials of the transformer one has to deal with a nonlinear problem. The occurring time-varying magnetic field induces eddy-currents in the tank walls and hence additional losses. In this work the parallel time periodic-finite element method (parallel TPFEM) [1], [2] and the harmonic balance fixed-point technique (HBFP) [3]-[6] are compared.

II. FEM Formulation and Modelling

A. Parallel Time-Periodic Finite-Element Method

In case of the parallel TPFEM method, applying Galerkin techniques to the differential equations resulting from the $\boldsymbol{A}$-V formulation, one obtains a system of nonlinear ordinary differential equations:

$$\boldsymbol{S}(\boldsymbol{x})+\boldsymbol{C}\frac{d}{dt}\boldsymbol{x}=\boldsymbol{f}\ . \qquad (1)$$

The matrix $\boldsymbol{S}$ is nonlinear due to its dependence on the unknown vector $\boldsymbol{x}$ and hence on μ, $\boldsymbol{C}$ is a constant coefficient matrix, $\boldsymbol{f}$ is the right-hand-side vector.

By considering time periodic conditions $\boldsymbol{x}_i=\pm\boldsymbol{x}_{i+n}$, all the nonlinear equations for one or half period are written as

$$\begin{cases}\boldsymbol{S}(\boldsymbol{x}_1)+\dfrac{1}{\Delta t}\boldsymbol{C}(\boldsymbol{x}_1\mp\boldsymbol{x}_n)=\boldsymbol{f}_1\\ \boldsymbol{S}(\boldsymbol{x}_2)+\dfrac{1}{\Delta t}\boldsymbol{C}(\boldsymbol{x}_2-\boldsymbol{x}_1)=\boldsymbol{f}_2\\ \qquad\vdots\\ \boldsymbol{S}(\boldsymbol{x}_n)+\dfrac{1}{\Delta t}\boldsymbol{C}(\boldsymbol{x}_n-\boldsymbol{x}_{n-1})=\boldsymbol{f}_n\end{cases}, \qquad (2)$$

where n is the number of time steps in a (half) period, Δt is the time interval, the subscript indicates the time step, and the signs − and + in (2) correspond to the ordinary and half time-periodic conditions, respectively. In the parallel TPFEM [1], [2], the large nonlinear system of equations (2) is solved by using parallel computing with pure message passing interface (MPI) programming.

Adopting the Newton-Raphson (NR) method as a nonlinear iteration method, the linearized equations of the TPFEM can be written as

$$\begin{bmatrix}\tilde{\boldsymbol{T}}_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} & \mp\tilde{\boldsymbol{C}}_n\\ -\tilde{\boldsymbol{C}}_1 & \tilde{\boldsymbol{T}}_2 & \cdots & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & -\tilde{\boldsymbol{C}}_2 & \ddots & \vdots & \vdots\\ \vdots & \vdots & \ddots & \ddots & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & -\tilde{\boldsymbol{C}}_{n-1} & \tilde{\boldsymbol{T}}_n\end{bmatrix}\begin{Bmatrix}\Delta\boldsymbol{x}_1\\ \vdots\\ \Delta\boldsymbol{x}_n\end{Bmatrix}=\begin{Bmatrix}-\boldsymbol{G}_1\\ \vdots\\ -\boldsymbol{G}_n\end{Bmatrix} \qquad (3)$$

and

$$\tilde{\boldsymbol{T}}_i=\boldsymbol{S}_i+\frac{\boldsymbol{C}}{\Delta t},\ \boldsymbol{C}_i=\frac{\boldsymbol{C}}{\Delta t},\ \boldsymbol{S}_i=\frac{\boldsymbol{S}(\boldsymbol{x}_i)}{\partial\boldsymbol{x}} \qquad (4)$$

where $\Delta\boldsymbol{x}_i$ is the increment of $\boldsymbol{x}_i$ and $\boldsymbol{G}_i$ is the residual. For solving the nonsymmetric linear system (3), we adopt the BiCGstab2 method and the localized ILU preconditioning.

B. Harmonic Balance Fixed-Point Method

In case of the harmonic balance fixed-point method [3], [4], applying Galerkin techniques to the differential equations resulting from the $\boldsymbol{T},\phi-\phi$ formulation, one obtains a system of nonlinear, ordinary differential equations of the form

$$\boldsymbol{S}(\rho)\boldsymbol{x}+\frac{d}{dt}\left[\boldsymbol{C}(\mu)\boldsymbol{x}\right]=\boldsymbol{f}\ . \qquad (5)$$

$\boldsymbol{S}$ depends on the resistivity ρ and is hence independent of $\boldsymbol{x}$ and time t. The mass matrix $\boldsymbol{C}$ depends on the permeability μ and hence on $\boldsymbol{x}$ and t. The vector $\boldsymbol{x}$ gathers the unknowns, $\boldsymbol{f}$ is the right hand side vector. Using the HBFP technique [5], the equation system becomes

$$\left[\boldsymbol{S}(\rho)+jm\omega\boldsymbol{C}(\mu_{FP}^{(s)})\right]\boldsymbol{X}_m^{(s+1)}=\mathscr{F}_m\left(jm\omega\boldsymbol{C}\left(\mu_{FP}^{(s)}-\mu^{(s)}\right)\boldsymbol{x}^{(s)}+\boldsymbol{f}\right), \qquad (6)$$

where $\mu_{FP}^{(s)}$ is the fixed-point reluctivity at the *s-th* iteration step and $\mathcal{F}_m$ denotes the *m-th* harmonic of the Fourier transform.

Since the nonlinearity is due to the dependence of $\boldsymbol{C}(\mu)$ on the solution, it is obvious to define a field independent fixed-point permeability μ_{FP}. Hence, one has to solve the given equation system in each iteration step s. The voltage excitation of the coils is implemented as described in [7].

III. NUMERICAL INVESTIGATIONS

To compare these two methods, a single phase transformer was modelled as a benchmark problem. The finite element models of the different methods should be as close as possible to each other. Half of the primary and of the secondary winding are wound around each limb with the two halves connected in series. In this approach the coils are assumed to be voltage driven. The main parameters of the transformer are given in TABLE I.

TABLE I
MAIN PARAMETERS OF THE TRANSFORMER

Values		Unit	Primary winding	Secondary winding
Voltage	U_{rms}	V	6600	210
Current	I_{rms}	A	3.03	95.2
Resistance	R	Ω	21.7	0.0163
No. of turns	N	1	1886	60
Frequency	f	Hz	50	

The parameters in the table are U_{rms} for the root mean square value of the excitation voltage, I_{rms} for the given current, R indicates the resistance of the windings, N is the number of turns and f is the operating frequency.

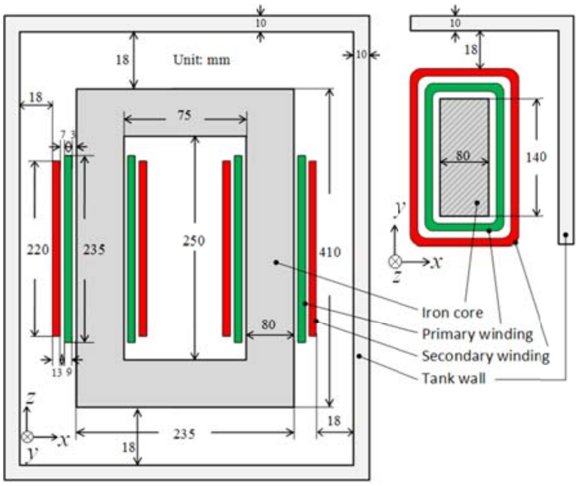

Fig. 1. Basic transformer model.

Fig. 1 shows the basic transformer design with the steel tank surrounding the transformer. If the excitation of the coils is sinusoidal, the resulting time varying magnetic field causes eddy currents in the highly permeable, conductive metal housing and hence additional losses. These eddy-current losses are to be investigated. Because of symmetry only an eighth of the problem domain is to be included in the finite element model.

FE-model parallel TPFEM

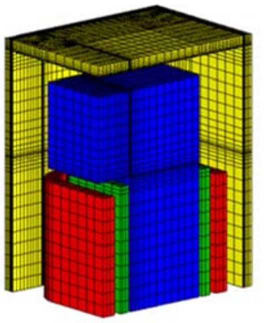
Fig. 2: Eighth of the transformer model for the parallel TPFEM

FE-model HBFP

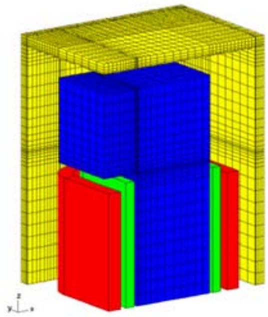
Fig. 3: Eighth of the transformer model for the HBFP

Fig. 2 and Fig. 3 show the finite element mesh of the problem domain. With the $\boldsymbol{A}$-V formulation used the coils are modelled with finite elements. With the $\boldsymbol{T},\phi-\phi$ formulation the coils need not to be included in the mesh. The nonlinearity of the core and tank material is considered by two different *B-H* characteristics.

IV. NUMERICAL RESULTS

Fig. 4 shows the normalized deviation of the computed currents of the coils when the coils are voltage driven.

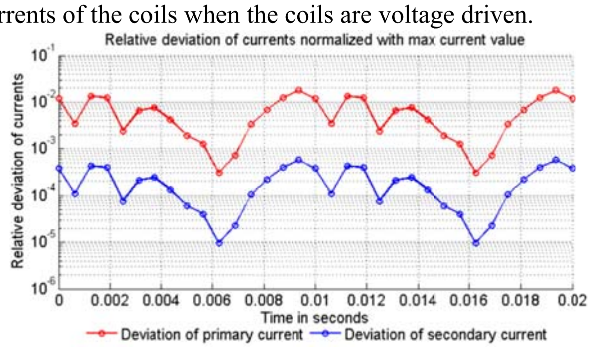

Fig. 4: Comparison of the calculated currents of the coils.

Further results will be presented at the conference and in the full paper.

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